

Employing spin symmetry to disentangle different models for the XYZ states

Martin Cleven,^{1,*} Feng-Kun Guo,^{2,†} Christoph Hanhart,^{3,‡} Qian Wang,^{3,§} and Qiang Zhao^{1,¶}

¹*Institute of High Energy Physics and Theoretical Physics Center for
Science Facilities, Chinese Academy of Sciences, Beijing 100049, China*

²*Helmholtz-Institut für Strahlen- und Kernphysik and Bethe Center
for Theoretical Physics, Universität Bonn, D-53115 Bonn, Germany*

³*Institut für Kernphysik and Institute for Advanced Simulation,
Forschungszentrum Jülich, D-52425 Jülich, Germany*

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In order to test different models proposed for some states discovered recently in the charmonium mass range that do not fit into the pattern predicted by the conventional quark model, we derive predictions for the spectrum within the hadro-charmonium picture, the tetraquark picture as well as the hadronic molecular approach. We exploit heavy quark spin symmetry for the hadro-charmonium and hadronic molecule scenarios. The patterns that emerge from the different models turn out to be quite distinct. For example, only within the hadro-charmonium picture a pseudoscalar state emerges that is lighter than the $Y(4260)$. Possible discovery channels of these additional states are discussed.

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I. INTRODUCTION

In the past decade a number of states in the charmonium and bottomonium sectors have been discovered with properties in conflict with the quark models for mesons which were quite successful in describing the low-lying heavy quarkonium states as $Q\bar{Q}$ ($Q = c, b$) states, such as the classical Godfrey–Isgur quark model [1] and Cornell potential model [2–4]. The newly observed structures in the heavy quarkonium mass region include, among many others, the $X(3872)$ [5], $Y(4260)$ [6] and the charged states $Z_c(3900)$ [7–9], $Z_c(4020)$ [10–13] and $Z_c(4430)$ [14–16] in the charmonium sector and the $Z_b(10610)$, $Z_b(10650)$ [17] in the bottomonium sector (see, e.g. the mini review on heavy

*Electronic address: cleven@ihep.ac.cn

†Electronic address: fkguo@hiskp.uni-bonn.de

‡Electronic address: c.hanhart@fz-juelich.de

§Electronic address: q.wang@fz-juelich.de

¶Electronic address: zhaoq@ihep.ac.cn

quarkonium spectroscopy in the Review of Particle Physics by the PDG [18]). Because of the failure in describing these structures simply as $Q\bar{Q}$ mesons, they are considered as candidates of exotic hadrons¹. Various models were proposed to explain these structures, see e.g. Ref. [20]. However, no consensus for almost any of these states has been achieved. It is thus of utmost importance to scrutinize these models, and manifest their distinct predictions. Because the near-threshold narrow structures in the continuum channel of the open-flavor meson pair cannot be explained by just a threshold cusp [21], we will assume that all the states to be discussed correspond to physical states. The dynamical structure of these states are being investigated in several different models. The purpose of this work is to present distinct predictions of several commonly discussed models. The scenarios to be considered include the tetraquark model, the hadro-charmonium model, and hadronic molecules. A brief description of each model including a list of relevant references and a short description of the underlying assumptions will be presented in the corresponding sections.

One has to keep in mind that different components mix with each other whenever such a mixing is not forbidden. Thus, when we present predictions for a certain scenario for a given state, we always mean that we are looking for the consequences that arise if that scenario is assumed to be dominant.

The key feature we are going to exploit in this work is heavy quark spin symmetry (HQSS). It is an approximate symmetry and becomes exact in the limit of infinitely heavy quarks. It arises because the spin-dependent quark-gluon coupling in quantum chromodynamics (QCD) is proportional to the magnetic moment of the heavy quark, and vanishes in the heavy quark limit. As a result, it leads to spin multiplets. Within a multiplet, the masses are degenerate in the heavy quark limit, and the mass splittings depend on the dynamics of the model for finite quark masses. As will be discussed, the multiplet structure differs in different scenarios, and thus provides invaluable information. We will compare the spectroscopy predicted by each of these three scenarios in the following sections. We close with a summary.

II. HADRO-QUARKONIUM

A. Assumptions

A hadro-quarkonium is a system with a compact heavy quarkonium embedded inside a cloud of light hadronic matter [22, 23]. This scenario was motivated by the fact that several charmonium-

¹ In fact, in the seminal quark model paper [19], Gell-Mann already mentioned the possibility of multiquark states.

like states were only observed in final states of a specific charmonium and light hadrons. Examples are the $Y(4260)$ discovered in $J/\psi\pi\pi$ [6], the $Z_c(4430)$ discovered in $\psi'\pi$ [24], the $Y(4360)$ and $Y(4660)$ observed in $\psi'\pi\pi$ [25, 26]. The recent BESIII observation of similar cross sections for $J/\psi\pi^+\pi^-$ and $h_c\pi^+\pi^-$ at 4.26 GeV and 4.36 GeV in e^+e^- collisions [7, 11] stimulated Li and Voloshin to generalize the hadro-charmonium model to include HQSS breaking and describe the $Y(4260)$ and $Y(4360)$ as a mixture of two hadro-charmonia [27]:

$$Y(4260) = \cos\theta\psi_3 - \sin\theta\psi_1, \quad Y(4360) = \sin\theta\psi_3 + \cos\theta\psi_1, \quad (1)$$

where $\psi_1 \sim (1^{+-})_{c\bar{c}} \otimes (0^{-+})_{q\bar{q}}$ and $\psi_3 \sim (1^{--})_{c\bar{c}} \otimes (0^{++})_{q\bar{q}}$ are the wave functions of the $J^{PC} = 1^{--}$ hadro-charmonia with a 1^{+-} and 1^{--} $c\bar{c}$ core charmonium, respectively. It was argued in Ref. [27] that ψ_3 contains predominantly a ψ' (rather than the ground state J/ψ) and that ψ_1 contains predominately an h_c . The decays into the $J/\psi\pi^+\pi^-$ then occur through de-exciting ψ' to J/ψ in the light hadronic matter. The strength is controlled by the so-called chromo-polarizability $\alpha_{\psi'J/\psi}$ (see Ref. [28]).

If we assume that the $J/\psi\pi^+\pi^-$ events seen in e^+e^- collisions at energies around 4.26 and 4.36 GeV are mainly from the decays of $Y(4260)$ and $Y(4360)$, the BESIII data imply that $\Gamma(Y(4260) \rightarrow J/\psi\pi\pi)$ is similar to $\Gamma(Y(4360) \rightarrow J/\psi\pi\pi)$. To achieve this pattern a mixing angle as large as $\theta \approx 40^\circ$ is needed [27]. Such a large angle translates into a small mass difference between the ψ_1 and ψ_3 hadro-charmonia since

$$\tan(2\theta) = \frac{2m_{13}}{m_{\psi_1} - m_{\psi_3}}, \quad (2)$$

which can be obtained from

$$\begin{pmatrix} m_{Y(4260)} & 0 \\ 0 & m_{Y(4360)} \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} m_{\psi_1} & m_{13} \\ m_{13} & m_{\psi_3} \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}. \quad (3)$$

A mixing angle of around 40° leads to $m_{\psi_3} \approx 4.30$ GeV, $m_{\psi_1} \approx 4.32$ GeV and a mixing amplitude $m_{13} \approx 50$ MeV.

B. Consequences

In addition to the interference patterns in the line shapes discussed in Ref. [27], what else does the proposal of the $Y(4260)$ and $Y(4360)$ as mixed hadro-charmonia imply? As mentioned in the Introduction, HQSS is quite useful in this respect. The binding force between the charmonium core and the surrounding light hadronic matter is due to the exchange of soft gluons. Because both the

charmonium and light hadronic matter are color singlets, at least two gluons need to be exchanged. The leading order (LO) interaction is due to exchanging two chromo-electric gluons [29]. It is important to notice that the heavy (anti-)quark spin decouples from such an interaction. Therefore, the LO interaction between the light hadronic matter with a particular heavy quarkonium, $[Q\bar{Q}]$, should be the same as that for the spin partner of that $[Q\bar{Q}]$ state. This means that a hadro-quarkonium should have spin partner(s) just as its core heavy quarkonium does.

Exchanging one chromo-electric and one chromo-magnetic gluon provides a P and CP odd force, and thus induces the mixing between two hadro-quarkonia with core heavy quarkonia of opposite P and CP. The mixing between the ψ_3 and ψ_1 states, which contain the ψ' (P = -, CP = +) and the h_c (P = +, CP = -), respectively, is such an example. Exchanging two chromo-magnetic gluons provides one source for splitting the masses for one spin multiplet of hadro-quarkonia. It is suppressed by $\mathcal{O}(\Lambda_{\text{QCD}}^2/m_Q^2)$ in comparison with the LO interaction, and gives a tiny correction ($\sim 4\%$ for hadro-charmonium and $\sim 1\%$ for hadro-bottomonium) to the potential energy and thus to the mass of the hadro-quarkonium. The mass splitting between the spin partners within the same multiplet of hadro-quarkonia is therefore given approximately by the mass splitting between the core heavy quarkonia with the next-to-leading spin symmetry violation controlled by mixing analogous to the one discussed above for the $Y(4260)$ and the $Y(4360)$. In fact, in Ref. [30], analogous to the question of interest, HQSS has been used to predict that the $Y(4660)$ as a $\psi' f_0(980)$ bound state [32] which may be regarded as a specific example of hadro-charmonium has a spin partner: an $\eta'_c f_0(980)$ bound state, with mass of around 4616 MeV.

From the discussion above it follows that the ψ_3 state has a spin partner characterized by the same light quark cloud with the ψ' in the core replaced by the η'_c . We may call this state $\eta_c[\eta'_c]$. It has $J^{PC} = 0^{-+}$ and a mass of around

$$m_{\psi_3} - (m_{\psi'} - m_{\eta'_c}) \approx 4.25 \text{ GeV}. \quad (4)$$

Similarly, replacing the h_c in the ψ_1 state by any of the χ_{cJ} states leads to three spin partners of ψ_1 . The quantum numbers of these states composed of $(J^{++})_{c\bar{c}} \otimes (0^{-+})_{q\bar{q}}$ are $J^{PC} = J^{-+}$. Their masses are listed in Table I and shown in Fig. 1 for illustration. We notice that there are two states with $J^{PC} = 0^{-+}$ in analogy to 1^{--} . As in the vector channel the small mass difference of the pure spin states of about 40 MeV introduces a sizable mixing. The relation of the mixing amplitude in this case can be related to that in the vector case by constructing an CP-odd operator with HQSS breaking

$$\mathcal{O}_{\text{mixing}} = \frac{1}{4} \langle \vec{\chi}^\dagger \cdot \vec{\sigma} J' \rangle + \text{h.c.} = \vec{h}_c^\dagger \cdot \vec{\psi}' + \sqrt{3} \chi_{c0}^\dagger \eta'_c + \text{h.c.}, \quad (5)$$

Composition	Label	J^{PC}	Mass (GeV)
$\psi' \otimes (0^{++})_{q\bar{q}}$	ψ_3	1^{--}	4.30
$\eta'_c \otimes (0^{++})_{q\bar{q}}$	$\eta_c[\eta'_c]$	0^{-+}	4.25
$h_c \otimes (0^{-+})_{q\bar{q}}$	ψ_1	1^{--}	4.32
$\chi_{c0} \otimes (0^{-+})_{q\bar{q}}$	$\eta_c[\chi_{c0}]$	0^{-+}	4.21
$\chi_{c1} \otimes (0^{-+})_{q\bar{q}}$	$\eta_{c1}[\chi_{c1}]$	1^{-+}	4.31
$\chi_{c2} \otimes (0^{-+})_{q\bar{q}}$	$\eta_{c2}[\chi_{c2}]$	2^{-+}	4.35

TABLE I: Masses and quantum numbers of hadro-charmonia in the spin multiplets of the $\psi_1 \sim h_c \otimes (0^{-+})_{q\bar{q}}$ and $\psi_3 \sim \psi' \otimes (0^{++})_{q\bar{q}}$ states.

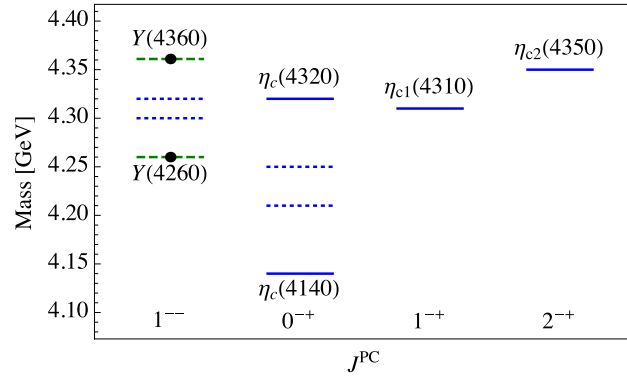


FIG. 1: The spectroscopy of the spin partners of $Y(4260)$ and $Y(4360)$ based on the assumption that they are mixed states of two 1^{--} hadro-charmonia as proposed in Ref. [27]. The dotted lines in the vector and pseudoscalar sectors represent the masses of the unmixed states. The masses of $Y(4360)$ and $Y(4260)$ were used as input for the analysis and are shown as dashed lines.

where $\vec{\sigma}$ are the Pauli matrices and $\langle \dots \rangle$ takes trace in the spinor space. The fields J' and $\vec{\chi}$ annihilates the S -wave and P -wave charmonium states, respectively, and are given explicitly by [31, 33]

$$J' = \vec{\psi}' \cdot \vec{\sigma} + \eta'_c, \quad \chi^i = \sigma^j \left(-\chi_{c2}^{ij} - \frac{1}{\sqrt{2}} \epsilon^{ijk} \chi_{c1}^k + \frac{1}{\sqrt{3}} \delta^{ij} \chi_{c0} \right) + h_c^i. \quad (6)$$

Thus the mixing amplitude is $\sqrt{3}m_{13}$,² up to corrections of order $(\Lambda_{\text{QCD}}/m_c)^2$, with $m_{13} \approx 50$ MeV. We can calculate the eigenvalues of the mass matrix³

$$m_{\eta_c(4320)} \approx 4.32 \text{ GeV}, \text{ and } m_{\eta_c(4140)} \approx 4.14 \text{ GeV}, \quad (7)$$

² We thank M. Voloshin for pointing this out.

³ Because these states have pseudoscalar quantum numbers, we name them as $\eta_c(\text{mass})$.

and the mixing angle is $\theta_0 \approx -38^\circ$ with θ_0 defined via

$$\eta_c(4320) = \cos \theta_0 \eta_c[\eta'_c] - \sin \theta_0 \eta_c[\chi_{c0}], \quad \eta_c(4140) = \sin \theta_0 \eta_c[\eta'_c] + \cos \theta_0 \eta_c[\chi_{c0}]. \quad (8)$$

Please observe that among the states predicted there is a state with exotic quantum numbers 1^{-+} : it appears to be a quite robust consequence of the hadro-charmonium scenario proposed in Ref. [27] that an η_{c1} state exists with a mass between those of $Y(4360)$ and $Y(4260)$.

Since all the considered hadro-charmonium states are above the corresponding thresholds for the decay into the core charmonium and two pions, we expect that they decay easily through dissociating the light hadronic matter into two pions, in complete analogy to $Y(4260)$ and $Y(4360)$ that were observed in final states consisting of a charmonium and two pions. This mechanism will introduce a width of ~ 100 MeV, in the ball park of the widths of the $Y(4260)$ and the $Y(4360)$, for each of them. Considering that both the widths of the η'_c and the χ_{c0} are about 10 MeV [18], much larger than those of the ψ' and the h_c , the predicted states $\eta_c(4140)$ and $\eta_c(4320)$ can also decay via the decays of the η'_c and χ_{c0} . Hadro-quarkonia can also decay into open flavor heavy meson and antiheavy meson pairs [34], but it is natural to expect the first mechanism to be dominant.

In the above, we have argued that if the $Y(4260)$ and the $Y(4360)$ are mixed hadro-charmonium states, then it is very likely that they have spin partners as shown in Fig. 1. Thus searching for these partners can provide valuable information on the nature of the $Y(4260)$ and $Y(4360)$. In which processes should they be searched for? As for the two pseudoscalar states $\eta_c(4140)$ and $\eta_c(4320)$, because of the probably sizeable mixing, both of them decay into the $\eta_c^{(\prime)} \pi \pi$ and $\chi_{c0} \pi \pi$. Being pseudoscalars, they cannot be produced directly in e^+e^- collisions. One way of searching for them is to measure the $\eta_c^{(\prime)} \pi^+ \pi^-$ invariant mass distribution for the decays $B^\pm \rightarrow K^\pm \eta_c^{(\prime)} \pi^+ \pi^-$ as suggested in Ref. [30] for searching for the spin partner of the $Y(4660)$. Another possible way of searching for the $\eta_c(4140)$ and $\eta_c(4320)$ is to study the radiative decays of the $Y(4260)$ and $Y(4360)$. Because the branching fraction for $\psi' \rightarrow \gamma \chi_{c0}$ is 10% [18], two orders of magnitude larger than that for $\psi' \rightarrow \gamma \eta'_c$, both the $\eta_c(4140)$ and $\eta_c(4320)$ states can be produced through the decays of the ψ_3 components of the $Y(4260)$ and $Y(4360)$ into their $\eta_c[\chi_{c0}]$ components. Hence, one may search for these two states in the process $e^+e^- \rightarrow \gamma \chi_{c0} \eta$ at the center-of-mass energies around the masses of the $Y(4260)$ and the $Y(4360)$.

The exotic state $\eta_{c1}(4310)$ and the state $\eta_{c2}(4350)$ can be searched for in analogous processes in the decays of $Y(4360)$ with the χ_{c0} in the final state replaced by the χ_{c1} and χ_{c2} , respectively. This kind of measurements may be performed at BESIII and a future high-luminosity super tau-charm factory.

If there are hadro-charmonium states, it is natural that the analogous hadro-bottomonium states should exist as well [23]. The interaction strength between the heavy quarkonium and the light hadronic matter is dictated by the chromo-polarizability which is a matrix element for the propagation of a color-octet $Q\bar{Q}$ pair [29], and thus depends on the wave functions of the heavy quarkonia involved. Numerically it was found that the off-diagonal chromo-polarizability for the transition $\Upsilon' \rightarrow \Upsilon\pi\pi$ is a factor of 3 smaller than that for the charmonium analogue $\psi' \rightarrow J/\psi\pi\pi$ [28].⁴ Therefore, there is no flavor symmetry connecting the hadro-bottomonium to the hadro-charmonium. However, because the mixing is induced by exchanging one chromo-electric and one chromo-magnetic gluon, one naively expects that the mixing amplitude for hadro-bottomonium states is much smaller than that for the hadro-charmonium states, and roughly scales down by a factor of m_c/m_b . If the unmixed states do not accidentally have a tiny mass difference, one should be able to neglect the mixing between hadro-bottomonium states. This means that the 1^{--} hadro-bottomonium states (possibly with a core $b\bar{b}$ of higher excitation [23]), should decay cleanly either into $\Upsilon(nS)\pi\pi$ or into $h_b(mP)\pi\pi$.

III. TETRAQUARK

A. Assumptions

Tetraquarks are four-quark states constructed in analogy to the regular quark model. In particular, the quarks are held together by effective gluon exchanges. Thus a necessary feature of each tetraquark model is that each isoscalar state is accompanied by (nearly) degenerate isovector states analogous to the ρ - ω degeneracy in the light meson sector.

Various variants for tetraquark models can be found in the literature. As one representative of this class of models we here discuss in detail only the implications of the most recently proposed interaction by Maiani et al. [36] (see also the review article Ref. [37]). Note that the interaction originally proposed for the hidden charm states by the same group [38] was shown to be inconsistent with the most recent discussions [36, 39]. In this model tetraquarks are understood as such

⁴ For both transitions, the dipion system is dominantly S -wave, and the available phase space for either of them is larger than 560 MeV for the $\pi\pi$ invariant mass. In this range, one expects that the $f_0(500)$, thus the $\pi\pi$ final state interaction (FSI), plays an important role. The FSI was neglected in Ref. [28]. However, it was found in Ref. [35] that the $\pi\pi$ FSI reduces the values of chromo-polarizability for the $c\bar{c}$ and $b\bar{b}$ by a similar factor. One should also keep in mind that the charged bottomonium-like $Z_b^\pm(10610)$ and $Z_b^\pm(10650)$ states couple to both the $\Upsilon\pi^\pm$ and $\Upsilon'\pi^\pm$, hence contribute to the dipion transition between Υ' and Υ . Such a contribution has not been taken into account in the analysis of the chromo-polarizability so far.

compact diquark–anti-diquark bound systems that the spin-spin interactions within the tetraquark are dominated by those within the diquarks.

In this model, the mass of a tetraquark is given by [36]

$$M = M_{00} + B_c \frac{\mathbf{L}^2}{2} - 2a \mathbf{L} \cdot \mathbf{S} + 2\kappa_{cq} [(\mathbf{s}_q \cdot \mathbf{s}_c) + (\mathbf{s}_{\bar{q}} \cdot \mathbf{s}_{\bar{c}})], \quad (9)$$

where \mathbf{s}_f ($f = q, c, \bar{q}, \bar{c}$) are the spins of (anti-)quarks, \mathbf{S} is the total spin, \mathbf{L} is the orbital angular momentum between the diquark and anti-diquark. The (anti-)quarks within the (anti-)diquarks are assumed to be in an S -wave. The parameters M_{00} , B_c , a and κ_{cq} are to be fixed from experiment. Denoting the spin of the diquark and anti-diquark as $\mathbf{s} = \mathbf{s}_q + \mathbf{s}_c$ and $\bar{\mathbf{s}} = \mathbf{s}_{\bar{q}} + \mathbf{s}_{\bar{c}}$, respectively, the Hamiltonian can be evaluated for a given tetraquark state of total angular momentum J , denoted by $|s, \bar{s}; S, L\rangle_J$,

$$M = M_{00} + B_c \frac{L(L+1)}{2} + a[L(L+1) + S(S+1) - J(J+1)] + \kappa_{cq} [s(s+1) + \bar{s}(\bar{s}+1) - 3]. \quad (10)$$

For $J = 1$ the expression agree to Eq. (38) in Ref. [36]. Note that the parameters B_c , a and κ_{cq} are positive values, extracted from the experimental data by Maiani et al. [36]. As a result, the mass of the tetraquarks increases with increasing L and S , but decreases for growing J , which is a rather unusual feature.

B. Consequences

A general feature of tetraquark models is that a very rich spectroscopy emerges. In addition, there are always approximately degenerate isospin singlet and isospin triplet states, analogous to the case of the ρ and ω for the traditional $q\bar{q}$ mesons.

Following Ref. [36] we will discuss the implications of the above model for S -wave and P -wave tetraquark states only. The identification of some of the tetraquark levels with observed states was presented already in Ref. [36]. Here we extend this investigation by discussing all possible states with the mentioned quantum numbers.

For S -wave tetraquarks, since $L = 0$ and $J = S$ we can use $|s, \bar{s}\rangle_J$ to abbreviate $|s, \bar{s}; S, L\rangle_J$. Then it follows from Eq. (10) that there are three sets of tetraquark states, whose masses are

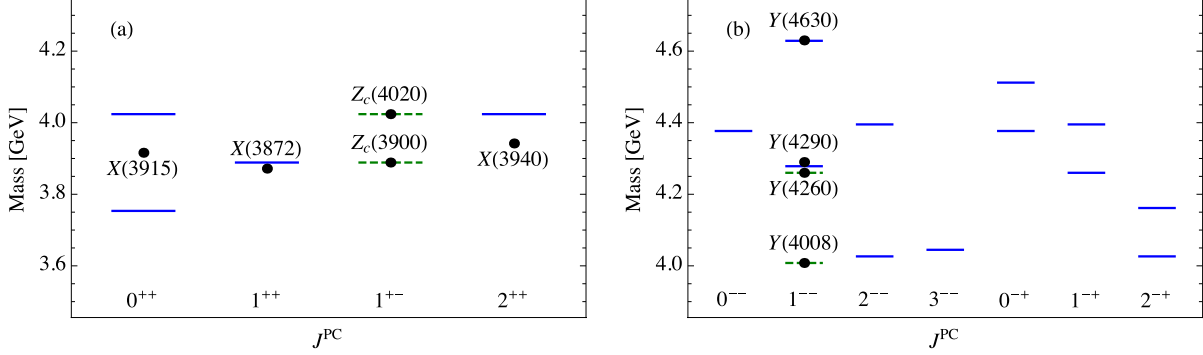


FIG. 2: The S -wave (a) and P -wave (b) tetraquark spectroscopy in the charmonium sector. The green dashed and blue solid lines are input and prediction, respectively.

approximately degenerate within the same set, since s and \bar{s} are equal:⁵

$$\begin{cases} 0^{++} : |0, 0\rangle_0 ; \\ 1^{++} : \frac{1}{\sqrt{2}} (|1, 0\rangle_1 + |0, 1\rangle_1) , & 1^{+-} : \frac{1}{\sqrt{2}} (|1, 0\rangle_1 - |0, 1\rangle_1) ; \\ J^{++} : |1, 1\rangle_0 , \quad |1, 1\rangle_2 , & 1^{+-} : |1, 1\rangle_1 . \end{cases} \quad (11)$$

For each of the J^{PC} quantum numbers, as mentioned before, the model predicts an isospin singlet as well as an isospin triplet.⁶ Therefore, there should be in total 24 S -wave tetraquark states already even without considering radial excitations that are expected about 400 MeV heavier than the corresponding ground states [36]. Among the ground states, the authors of Ref. [36] identified the $X(3872)$ as one of the neutral 1^{++} states and the $Z_c(3900)$ and $Z_c(4020)$ as the iso-triplet 1^{+-} states, $(|1, 0\rangle_1 - |0, 1\rangle_1) / \sqrt{2}$ and $|1, 1\rangle_1$, respectively. One may also assign $X(3915)$ and $X(3940)$ as $|1, 1\rangle_0$ and $|1, 1\rangle_2$, respectively [36], although for each of them there is quite a large deviation between the mass of the tetraquark predicted and the actual mass of the observed state, cf. Fig. 2. Therefore, at least 15 of the 24 S -wave tetraquarks are waiting for an observation.

For P -wave tetraquarks, four isospin singlet 1^{--} states without radial excitation were discussed in Ref. [36]. But there are many more states — a few with exotic quantum numbers (that can not

⁵ The quantum numbers are J^{PC} , where the C -parity is given for the iso-singlet and the neutral member of the iso-triplet.

⁶ If we take into account the strange and anti-strange quarks, there would be a nonet for each J^{PC} .

be reached by the conventional $\bar{q}q$ states) like 0^{--} and 1^{-+}

$$\left\{ \begin{array}{ll} 1^{--} : |0, 0; 0, 1\rangle_1 ; \\ 0^{-+} : \frac{1}{\sqrt{2}}(|1, 0; 1, 1\rangle_0 + |0, 1; 1, 1\rangle_0) , & 0^{--} : \frac{1}{\sqrt{2}}(|1, 0; 1, 1\rangle_0 - |0, 1; 1, 1\rangle_0) ; \\ 1^{-+} : \frac{1}{\sqrt{2}}(|1, 0; 1, 1\rangle_1 + |0, 1; 1, 1\rangle_1) , & 1^{--} : \frac{1}{\sqrt{2}}(|1, 0; 1, 1\rangle_1 - |0, 1; 1, 1\rangle_1) ; \\ 2^{-+} : \frac{1}{\sqrt{2}}(|1, 0; 1, 1\rangle_2 + |0, 1; 1, 1\rangle_2) , & 2^{--} : \frac{1}{\sqrt{2}}(|1, 0; 1, 1\rangle_2 - |0, 1; 1, 1\rangle_2) ; \\ 1^{--} : |1, 1; 0, 1\rangle_1 ; \\ 0^{-+} : |1, 1; 1, 1\rangle_0 ; \\ 1^{-+} : |1, 1; 1, 1\rangle_1 , & 2^{--} : |1, 1; 2, 1\rangle_2 ; \\ 2^{-+} : |1, 1; 1, 1\rangle_2 ; \\ 1^{--} : |1, 1; 2, 1\rangle_1 ; \\ 3^{--} : |1, 1; 2, 1\rangle_3 . \end{array} \right. \quad (12)$$

According to Eq. (10) the states in each line are (approximately) degenerate. In addition, as before, all of them appear as an isospin singlet and an isospin triplet when we restrict the light quark and anti-quark to be the up and down flavors. Thus, the above list amounts to 56 P -wave tetraquark states even without radial excitations taken into account.

The assignments of the 1^{--} P -wave tetraquark states to the observed structures in Ref. [36] are as follows: among the four 1^{--} states, three of them were identified with the $Y(4008)$, $Y(4260)$ and $Y(4630)$,⁷ and the other one was identified as one of two structures, called $Y(4220)$ ⁸ and $Y(4290)$ observed in $e^+e^- \rightarrow h_c\pi^+\pi^-$ [43]. The states $Y(4360)$ and $Y(4660)$ were suggested to be the radial excitations of the $Y(4008)$ and $Y(4260)$. Thus, up to the first radial excitation, only 6 of 112 (28 if considering only the isospin singlet ones) P -wave tetraquark states have candidates so far.

There is one salient feature of Eq. (10): for states with the same s , \bar{s} , S and L , the mass decreases for increasing J , which appears to be a consequence of the negative sign in front of $J(J+1)$. For instance, among the states $|1, 1; 2, 1\rangle_J$ with J^{--} , the 1^{--} state has the largest mass while the 3^{--} one has the smallest. Thus the observation of a rather light charmonium with $J = 3$ could provide strong support for the tetraquark picture of Ref. [36].

Besides the model discussed in detail above, also other tetraquark models can be found in the

⁷ It was proposed in the literature that $Y(4630)$ observed in the $\Lambda_c\bar{\Lambda}_c$ and $Y(4660)$ observed in $\psi'\pi^+\pi^-$ correspond to the same state [40, 41].

⁸ It was proposed in the literature that $Y(4220)$ observed $h_c\pi\pi$ and $Y(4260)$ observed in $J/\psi\pi\pi$ correspond to the same state [27, 42].

literature that differ in the underlying assumptions. For example, in Refs. [44–48] the states are treated as four-quark systems without any clustering into diquark–anti-diquark assumed. As a result, the color part of the wave functions includes both antitriplet and triplet or sextet and anti-sextet configurations for the quark and antiquark pairs, respectively [48]. As a result the number of the predicted S -wave tetraquarks (Fig. 2 of Ref. [48]) is twice as large as that of Maiani et al. Although this picture can explain certain phenomena such as the narrow width of $X(3872)$ due to its tiny $J/\psi + \rho$ and $J/\psi + \omega$ component in the wave function [45], $Y(4140)$ as the hidden strange analog $c\bar{c}s\bar{s}$ of $X(3872)$ [48], and $Z_b(10610)$ and $Z_b(10650)$ as $b\bar{b}q\bar{q}$ four-quark systems [46], there is an even larger number of tetraquarks waiting to be observed within this approach.

IV. HADRONIC MOLECULES

A hadronic molecule is an extended object that results from non-perturbative scatterings of two or more hadrons. The hadronic molecules of interest here are bound states of a pair of charmed or bottomed mesons, which are similar to the deuteron as a bound state of the proton and neutron [49]. Since the masses of some of the X, Y, Z states are close to S -wave thresholds and couple strongly to the corresponding continuum states, they are good candidates for hadronic molecules. For example, the $X(3872)$ is proposed to be a $D\bar{D}^* + c.c.$ molecule (see Refs. [50, 51] and many further studies in the literature), the $Y(4260)$ to be a $D_1\bar{D} + c.c.$ molecule [52, 53], the $Y(4360)$ to be a $D_1\bar{D}^* + c.c.$ molecule [54, 55], and the two charged states, $Z_b(10610)^\pm$ and $Z_b(10650)^\pm$, to be $B\bar{B}^* + c.c.$, $B^*\bar{B}^*$ molecules, respectively [56, 57].

Naively, one might expect that the number of possible molecules is at least as large as that of the available S -wave thresholds. In addition, since the open-charm and open-bottom mesons carry isospin $1/2$ a pair of them can couple to both isospin 0 and 1. One might therefore expect almost degenerate isoscalar and isovector states for each quantum number similar to the tetraquark scenario. However, both expectations are not correct. First of all, a shallow bound state with an unstable constituent can in general not be narrower than that constituent, but will typically be broader [58, 59]. In addition, the life time of a broad hadron, whose width is of the order of or even larger than the inverse of the range of forces, is too short to form a bound state with another hadron [60]. Thus, only the narrow $D_1(2420)$ with a width of ~ 25 MeV can form an observable hadronic molecule (examples will be discussed below), while the broad $D_1(2430)$ with a width of ~ 380 MeV cannot. In this sense it also appears natural that the widths of $Y(4260)$ and $Y(4360)$ are of order 100 MeV.

In addition, the scattering potential in general comprises two contributions: a long-ranged part mediated by one-pion exchange and a short-ranged part that is often parametrized as contact interactions (and that one might be phenomenologically identified with the exchange of heavier mesons, and could also come from s -channel $\bar{q}q$ states or more complicated dynamics). The short-ranged part needs to be fixed from data, as done in Refs. [61–63] for the systems of a pair of S -wave heavy and anti-heavy mesons using the information of the $X(3872)$ and $Z_b(10610)$ states as input. Since at present there is not enough experimental information available for doing this for all channels, in this Section we have to restrict ourselves to qualitative statements regarding the hadronic molecular picture.⁹ However, this has already allowed us to highlight some striking differences in the features of this model in comparison with the tetraquarks and hadro-charmonia.

As a guidance for the existence of shallow bound hadronic molecules (here, we wish to use the phrase “shallow” in a loose sense meaning states with binding energies significantly less than 100 MeV), we will study the contribution of the one-pion exchange and argue that if the one-pion exchange is repulsive for a given system the appearance of a bound state is unlikely, while a bound state could exist for an attractive one-pion exchange.¹⁰ This kind of argument is justified by the very small mass of the pion together with the observation that an attractive interaction mediated by a massless exchange particle always binds, regardless how weak the interaction is [87]. We also notice the argument by Eriscon and Karl [88] suggesting that two hadrons with an attractive one-pion exchange potential should form hadronic molecules if their reduced mass is sufficiently large. Based on this kind of reasoning it was possible to predict the existence of the $X(3872)$ well before its observation [89]. There are also examples discussed below where the one-pion exchange does not contribute and possible molecular states must then be bound either by coupled channel effects or by short-ranged interactions. In such a case, the reasoning used in this paper cannot be applied.

As mentioned above when talking about bound systems of mesons only those meson pairs are relevant where both mesons are sufficiently narrow. Here we focus on the ground state open charm meson doublet D and D^* (characterized by a charm quark and a light anti-quark contribution

⁹ For the discussion of the one-pion exchange in effective field theories for the $X(3872)$, see Refs. [61, 64–69]. In addition, there exist various model calculations based on one-meson exchange potential or $SU(4)$ extension of the light meson interactions. For the systems of a pair of S -wave heavy mesons, see, e.g., Refs. [70–83]. For the systems of an S -wave and a P -wave heavy meson, see Refs. [52, 84–86].

¹⁰ Note that in Ref. [82] it is claimed that one-pion exchange does not contribute to the binding of, e.g., $Z_c(3900)$, since it gets cancelled by the contribution of the η and η' exchanges in the $U(3)$ limit. However, especially in the $D^*\bar{D}$ system where the exchanged pion is near on-shell while η and η' are far off-shell, one should expect sizable violations of $U(3)$ symmetry.

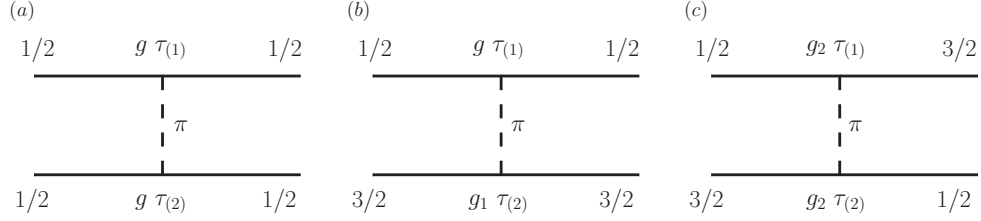


FIG. 3: The various one-pion exchange diagrams relevant for the hadronic molecular scenario. Diagrams (a) and (b) show the t -channel pion exchange for the $1/2 + 1/2$ and the $1/2 + 3/2$ system, respectively, while diagram (c) shows the u -channel exchange, relevant for the $1/2 + 3/2$ system only.

with $s_\ell^P = \frac{1}{2}^-$ where s_ℓ is the total angular momentum of the light part) and the spin doublet that contains $D_1(2420)$ and $D_2(2460)$ (characterized by a charm quark and a light anti-quark contribution with total $s_\ell^P = \frac{3}{2}^+$) and their anti-particles as possible constituents. In particular, we discuss the $\frac{1}{2} + \frac{1}{2}$ and $\frac{1}{2} + \frac{3}{2}$ hadronic molecules, where we have used the total angular momentum of the light quark contribution to characterize the states.

The Feynman diagram of the one-pion exchange is shown in Fig. 3. Because the pions are in the adjoint representation of the isospin $SU(2)$ group, and the non-strange heavy mesons are isospin-1/2 states, each heavy-meson-pion vertex is accompanied by a factor $\vec{\tau}$ which stands for the Pauli matrices operating in the isospin space. The full one-pion exchange contribution between two heavy mesons therefore comes with a factor $\vec{\tau}_{(1)} \cdot \vec{\tau}_{(2)}$, where the subindices label the heavy meson to which the corresponding vertex is attached. One finds for a given pair of isospin-1/2 particles with total isospin I and the third component I_3

$$\langle II_3 | \vec{\tau}_{(1)} \cdot \vec{\tau}_{(2)} | II_3 \rangle = 2 \left[I(I+1) - \frac{3}{2} \right], \quad (13)$$

which is -3 for $I = 0$ and 1 for $I = 1$. Thus, if for a given set of quantum numbers the one-pion exchange contribution is attractive in the isoscalar channel, it will be repulsive in the isovector, and vice versa. Consequently, there is either an isoscalar state *or* an isovector state with some fixed quantum numbers, but typically not both. This is in contrast to the predictions for the tetraquark models discussed in the previous section, since the one-gluon exchange potential is flavor independent.

In this paper we discuss where to expect S -wave molecular states from non-perturbative interactions of the $\frac{1}{2}^-$ multiplet (D, D^*) with the anti-particles (\bar{D}, \bar{D}^*) or with the $\frac{3}{2}^+$ multiplet (\bar{D}_1, \bar{D}_2). Particles and anti-particles need to be combined such that the states have a well defined C -parity. Bound systems formed from members of two $\frac{3}{2}^+$ multiplets are potentially too broad to

show a striking signal in experiment and will therefore not be discussed here. Within this model space the following quantum numbers can be reached for $\frac{1}{2} + \frac{1}{2}$,

$$\begin{aligned}
0^{++} &: |D^*\bar{D}^*\rangle, \quad |D\bar{D}\rangle; \\
1^{+-} &: \frac{1}{\sqrt{2}}|D\bar{D}^* - \bar{D}D^*\rangle, \quad |D^*\bar{D}^*\rangle; \\
1^{++} &: \frac{1}{\sqrt{2}}|D\bar{D}^* + \bar{D}D^*\rangle; \\
2^{++} &: |D^*\bar{D}^*\rangle.
\end{aligned} \tag{14}$$

Here and in the following, we use the phase convention for the charge conjugation so that $\bar{D}_J = \mathcal{C}D_J\mathcal{C}^{-1}$ with D_J representing any charmed meson and \mathcal{C} the charge conjugation operator. Whenever there appears more than one state for given quantum numbers, hadronic molecular states may appear as a result of coupled-channel dynamics and very limited statements are possible without a detailed dynamical calculation. Exception to this are the 1^{++} and 2^{++} states. It is natural to identify the 1^{++} state with the $X(3872)$. In addition, it turned out that HQSS forces the binding potentials for these two cases to be equal at LO in the low-energy expansion such that one arrives at the prediction of a tensor state X_2 located close to the $D^*\bar{D}^*$ threshold [61, 63]. In Ref. [90] it was shown that within a scheme where the one-pion exchange is treated perturbatively, this result is stable under inclusion of the $D\bar{D}$ inelastic channel in the D wave. As mentioned above, as a result of the isospin factor of the one-pion exchange, in the molecular picture one does not expect any isovector states with the quantum numbers 1^{++} and 2^{++} .

Coupled-channel equations based on LO S -wave interactions typically produce as many states as the channels included in the calculation. Thus, we expect two states to be present with quantum numbers 1^{+-} that strongly couple to $D\bar{D}^*$ and $D^*\bar{D}$. Indeed, both are established experimentally with the isovector states $Z_c(3900)$ and $Z_c(4020)$. Current experimental evidence locates both states above the thresholds¹¹. For S -wave states this can be achieved either by momentum dependent interactions [91] or non-trivial coupled-channel dynamics. Which one of these mechanisms is at work here (if any) requires a detailed model-building which however is beyond the scope of this work. Again, since isovector states are observed one should not expect their isoscalar partner states with 1^{+-} quantum numbers.

The situation for 0^{++} is more complicated, since also for this system we are faced with a coupled-channel system. In addition, the one-pion exchange is not allowed for the diagonal $D\bar{D}$ interaction

¹¹ It should be stressed that the location of the poles is not settled yet: that the peaks in the data are located above the threshold does not necessarily imply that the poles are above the threshold as well. This is demonstrated for the Z_b -states in Ref. [57].

as a consequence of parity conservation. Neither does HQSS equalize the LO contact interaction in this channel to that in any other channel [61]. Thus, at this point we cannot make any statement about the existence or non-existence of hadronic molecular states in the 0^{++} channel.

The number of available channels and quantum numbers for the $\frac{1}{2} + \frac{3}{2}$ system is even much larger than the one for $\frac{1}{2} + \frac{1}{2}$:

$$\begin{aligned}
1^{-+} : & \quad \frac{1}{\sqrt{2}}|D\bar{D}_1 + D_1\bar{D}\rangle, \quad \frac{1}{\sqrt{2}}|D^*\bar{D}_1 + D_1\bar{D}^*\rangle, \quad \frac{1}{\sqrt{2}}|D^*\bar{D}_2 + D_2\bar{D}^*\rangle; \\
1^{--} : & \quad \frac{1}{\sqrt{2}}|D\bar{D}_1 - D_1\bar{D}\rangle, \quad \frac{1}{\sqrt{2}}|D^*\bar{D}_1 - D_1\bar{D}^*\rangle, \quad \frac{1}{\sqrt{2}}|D^*\bar{D}_2 - D_2\bar{D}^*\rangle; \\
2^{-+} : & \quad \frac{1}{\sqrt{2}}|D\bar{D}_2 + D_2\bar{D}\rangle, \quad \frac{1}{\sqrt{2}}|D^*\bar{D}_1 + D_1\bar{D}^*\rangle, \quad \frac{1}{\sqrt{2}}|D^*\bar{D}_2 + D_2\bar{D}^*\rangle; \\
2^{--} : & \quad \frac{1}{\sqrt{2}}|D\bar{D}_2 - D_2\bar{D}\rangle, \quad \frac{1}{\sqrt{2}}|D^*\bar{D}_1 - D_1\bar{D}^*\rangle, \quad \frac{1}{\sqrt{2}}|D^*\bar{D}_2 - D_2\bar{D}^*\rangle; \\
0^{-+} : & \quad \frac{1}{\sqrt{2}}|D^*\bar{D}_1 + D_1\bar{D}^*\rangle; \\
0^{--} : & \quad \frac{1}{\sqrt{2}}|D^*\bar{D}_1 - D_1\bar{D}^*\rangle; \\
3^{-+} : & \quad \frac{1}{\sqrt{2}}|D^*\bar{D}_2 + D_2\bar{D}^*\rangle; \\
3^{--} : & \quad \frac{1}{\sqrt{2}}|D^*\bar{D}_2 - D_2\bar{D}^*\rangle.
\end{aligned} \tag{15}$$

In this case, one can also check in which channels HQSS predicts the same LO interaction. For each isospin, 0 or 1, there are four independent interactions denoted as $\langle s_{\ell 1}, s_{\ell 2}, s_L | \hat{V}_I | s'_{\ell 1}, s'_{\ell 2}, s_L \rangle$ with $s_L = s_{\ell 1} + s_{\ell 2} = s'_{\ell 1} + s'_{\ell 2}$:¹²

$$\left\langle \frac{1}{2}, \frac{3}{2}, 1 \left| \hat{V}_I \right| \frac{1}{2}, \frac{3}{2}, 1 \right\rangle, \left\langle \frac{1}{2}, \frac{3}{2}, 1 \left| \hat{V}_I \right| \frac{3}{2}, \frac{1}{2}, 1 \right\rangle, \left\langle \frac{1}{2}, \frac{3}{2}, 2 \left| \hat{V}_I \right| \frac{1}{2}, \frac{3}{2}, 2 \right\rangle, \left\langle \frac{1}{2}, \frac{3}{2}, 2 \left| \hat{V}_I \right| \frac{3}{2}, \frac{1}{2}, 2 \right\rangle. \tag{16}$$

It turns out that for the diagonal interactions all of the above listed channels have a different linear combination of the matrix elements given in Eq. (16). Therefore, not much can be derived from HQSS without further input in this case.

In addition, in case of the $\frac{1}{2} + \frac{3}{2}$ system two kinds of one-pion exchange contributions are possible in the channels with odd parity. They correspond to the t -channel and u -channel one-pion exchange, cf. Fig. 3. Denoting the coupling constants for the coupling of a pion with two $\frac{1}{2}$ states (P -wave), with two $\frac{3}{2}$ states (P -wave) and connecting a $\frac{1}{2}$ state and a $\frac{3}{2}$ state (D -wave) by g , g_1 and g_2 , respectively, the potential for the t -channel exchange is proportional to gg_1 and the one for the u -channel exchange is proportional to g_2^2 . While the magnitude and sign of the coupling constant g are fixed by data and lattice QCD calculations, nothing is known yet about g_1 .

¹² We thank J. Nieves for pointing this out.

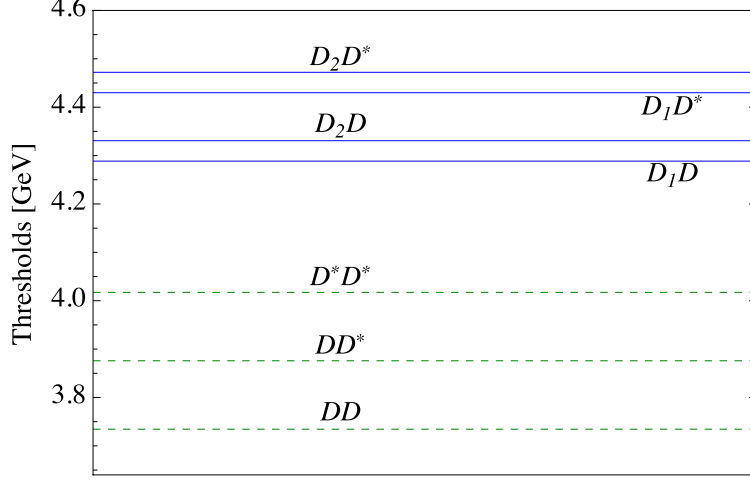


FIG. 4: The two-body thresholds in the charmonium mass range potentially relevant for the formation of hadronic molecules.

Therefore, we cannot identify the channels where the corresponding one-pion exchange potential is attractive. The situation is different for the part of u -channel exchange with two D -wave vertices proportional to g_2^2 (although the additional momentum factors on the vertices might suppress this part of the potential). Looking only at the sign of this u -channel exchange contribution for the uncoupled channels, one finds attraction for isoscalar states with the exotic quantum numbers 0^{--} and 3^{-+} , while this part of the interaction is attractive in the isovector channel with the quantum numbers 0^{-+} and 3^{--} . However, as mentioned above, since for these systems we cannot even analyze the complete one-pion exchange contribution no strong conclusion can be drawn regarding the existence or non-existence of these states.

For the quantum numbers $1^{-\pm}$, $2^{-\pm}$ three channels couple to each other. Thus, here even less can be said without dynamical analysis. However, some general statements are still possible. For instance, since hadronic molecular states appear in the vicinity of S -wave thresholds, the lightest molecular state in the $\frac{1}{2} + \frac{3}{2}$ family is expected to be close to the lowest threshold, namely $D_1\bar{D}$ (cf. Fig. 4), and with the quantum numbers of either 1^{--} or 1^{-+} . Indeed, in Refs. [52, 53] the $Y(4260)$ was proposed to be (predominantly) a $D_1\bar{D}$ bound state with $J^{PC} = 1^{--}$, and in Ref. [92] the molecular states with exotic quantum numbers $J^{PC} = 1^{-+}$ were investigated.

For the quantum numbers $0^{-\pm}$, if a corresponding state exists at all, it should be around the $D^*\bar{D}_1$ threshold and thus more than 100 MeV above the mass of the $Y(4260)$. While this mass range is similar to what is predicted in the tetraquark picture, it is very different to what is predicted in the hadro-charmonium scenario, where the 0^{-+} states are predicted to be the lightest

of their kind.

One striking distinction between the tetraquark picture and the hadronic molecular scenario becomes visible for the $J = 3$ states: while in the tetraquark model of Ref. [36] those are amongst the lightest states, in the hadronic molecular picture, if they exist, they should be close to the $D_2\bar{D}^*$ threshold, again more than 100 MeV above the $Y(4260)$, cf. Fig. 4.

V. SUMMARY

In this work we have investigated the exotic charmonium spectrum in different scenarios based on HQSS. In particular, we have compared the spectra arising from three models, i.e. hadro-charmonium, tetraquark and hadronic molecule, which turn out to provide quite distinct predictions for not yet discovered “ XYZ ” states.

In the hadro-charmonium scenario we find that if $Y(4260)$ and $Y(4360)$ are mixed hadro-charmonium states with $J^{PC} = 1^{--}$ as proposed in Ref. [27] then two spin partners with $J^{PC} = 0^{-+}$ and two more with $J^{PC} = 1^{-+}$ and $J^{PC} = 2^{-+}$, respectively, should exist as well. Their possible production and decay modes are discussed to guide the experimental search in the future.

In the tetraquark scenario the spectrum is much richer than in other scenarios with approximate degeneracies between isospin singlet and triplet states. If one assigns the observed $X(3872)$, $Z_c(3900)$ and $Z_c(4020)$ to the S -wave states, some of the states with $J^{PC} = 1^{--}$ can also be assigned to the P -wave tetraquark states, i.e. $Y(4008)$, $Y(4260)$ and $Y(4630)$, while the possible existence of two 1^{--} states, $Y(4220)$ and $Y(4290)$ in $e^+e^- \rightarrow h_c\pi^+\pi^-$ will accommodate all the P -wave 1^{--} tetraquark ground states. However, it should be noted that so far only a small number of predicted tetraquark states can be assigned to existing observations — at least 67 among 80 of the ground states are left to be discovered, if compact tetraquarks provide the dominant components of the new XYZ states.

In the scenario of hadronic molecule very little can be said without detailed modeling of the relevant hadron-hadron potentials. Under the assumption that the one-pion exchange potential plays a dominant role, one expects bound states to appear either in the isoscalar or isovector channel but not in both simultaneously. The observation of the isospin singlet $X(3872)$ and the isospin triplet $Z_c(3900)$ and $Z_c(4020)$ without evidence of isospin partners matches this expectation well. Taking into account that the constituent mesons are to be narrow enough in the hadronic molecular system, the allowed number for hadronic molecules formed by low-lying narrow charmed meson pairs turns out to be a lot smaller than what is predicted within the tetraquark scenario.

In order to highlight that the spectroscopy of the different scenarios is indeed very different we summarize the implications for two quantum numbers: Amongst the three models only the hadro-charmonium scenario predicts 2 0^{-+} states with one of them being even lighter than the $Y(4260)$. In the tetraquark picture also two 0^{-+} states appear, however, both are predicted to be heavier than $Y(4260)$. Due to lack of a dynamical model, in the hadronic molecular picture there is no prediction yet for a 0^{-+} state, however, if it exists it should be located near the $D_1\bar{D}^*$ threshold — again above the mass of the $Y(4260)$.

Interestingly, we find that the tetraquark model of Ref. [36] and hadronic molecular scenarios predict strikingly different patterns for the $J = 3$ states. In the former scenario the $J^{PC} = 3^{--}$ state is expected to be among the lightest states, while in the hadronic molecular scenario the $J = 3$ states should have masses close to the $D_2\bar{D}^*$ threshold if they exist. An observation of the $J = 3$ states should provide crucial information about the underlying dynamics.

It is very important to test the different scenarios in future experimental studies at LHCb, BESIII, Belle-II, PANDA and others.

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- [1] S. Godfrey and N. Isgur, Phys. Rev. D **32**, 189 (1985).
 - [2] E. Eichten, K. Gottfried, T. Kinoshita, J. B. Kogut, K. D. Lane and T. M. Yan, Phys. Rev. Lett. **34**, 369 (1975) [Phys. Rev. Lett. **36**, 1276 (1976)].
 - [3] E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane and T. M. Yan, Phys. Rev. D **17**, 3090 (1978) [Phys. Rev. D **21**, 313 (1980)].
 - [4] E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane and T. M. Yan, Phys. Rev. D **21**, 203 (1980).
 - [5] S. K. Choi *et al.* [Belle Collaboration], Phys. Rev. Lett. **91**, 262001 (2003) [hep-ex/0309032].
 - [6] B. Aubert *et al.* [BaBar Collaboration], Phys. Rev. Lett. **95**, 142001 (2005) [hep-ex/0506081].

- [7] M. Ablikim *et al.* [BESIII Collaboration], Phys. Rev. Lett. **110**, 252001 (2013) [arXiv:1303.5949 [hep-ex]].
- [8] Z. Q. Liu *et al.* [Belle Collaboration], Phys. Rev. Lett. **110**, 252002 (2013) [arXiv:1304.0121 [hep-ex]].
- [9] T. Xiao, S. Dobbs, A. Tomaradze and K. K. Seth, Phys. Lett. B **727**, 366 (2013) [arXiv:1304.3036 [hep-ex]].
- [10] M. Ablikim *et al.* [BESIII Collaboration], Phys. Rev. Lett. **112**, 022001 (2014) [arXiv:1310.1163 [hep-ex]].
- [11] M. Ablikim *et al.* [BESIII Collaboration], Phys. Rev. Lett. **111**, 242001 (2013) [arXiv:1309.1896 [hep-ex]].
- [12] M. Ablikim *et al.* [BESIII Collaboration], Phys. Rev. Lett. **112**, 132001 (2014) [arXiv:1308.2760 [hep-ex]].
- [13] M. Ablikim *et al.* [BESIII Collaboration], Phys. Rev. Lett. **113**, 212002 (2014) [arXiv:1409.6577 [hep-ex]].
- [14] S. K. Choi *et al.* [Belle Collaboration], Phys. Rev. Lett. **100**, 142001 (2008).
- [15] K. Chilikin *et al.* [Belle Collaboration], Phys. Rev. D **88**, 074026 (2013) [arXiv:1306.4894 [hep-ex]].
- [16] R. Aaij *et al.* [LHCb Collaboration], Phys. Rev. Lett. **112**, 222002 (2014) [arXiv:1404.1903 [hep-ex]].
- [17] A. Bondar *et al.* [Belle Collaboration], Phys. Rev. Lett. **108**, 122001 (2012) [arXiv:1110.2251 [hep-ex]].
- [18] K. A. Olive *et al.* [Particle Data Group], Chin. Phys. C **38**, 090001 (2014).
- [19] M. Gell-Mann, Phys. Lett. **8**, 214 (1964).
- [20] N. Brambilla *et al.*, Eur. Phys. J. C **71**, 1534 (2011) [arXiv:1010.5827 [hep-ph]].
- [21] F.-K. Guo, C. Hanhart, Q. Wang and Q. Zhao, Phys. Rev. D **91**, 051504 (2015) [arXiv:1411.5584 [hep-ph]].
- [22] M. B. Voloshin, Prog. Part. Nucl. Phys. **61**, 455 (2008) [arXiv:0711.4556 [hep-ph]].
- [23] S. Dubynskiy and M. B. Voloshin, Phys. Lett. B **666**, 344 (2008) [arXiv:0803.2224 [hep-ph]].
- [24] R. Mizuk *et al.* [Belle Collaboration], Phys. Rev. D **80**, 031104 (2009) [arXiv:0905.2869 [hep-ex]].
- [25] X. L. Wang *et al.* [Belle Collaboration], Phys. Rev. Lett. **99**, 142002 (2007) [arXiv:0707.3699 [hep-ex]].
- [26] B. Aubert *et al.* [BaBar Collaboration], Phys. Rev. Lett. **98**, 212001 (2007) [hep-ex/0610057].
- [27] X. Li and M. B. Voloshin, Mod. Phys. Lett. A **29**, 1450060 (2014) [arXiv:1309.1681 [hep-ph]].
- [28] A. Sibirtsev and M. B. Voloshin, Phys. Rev. D **71**, 076005 (2005) [hep-ph/0502068].
- [29] M. B. Voloshin, Mod. Phys. Lett. A **19**, 665 (2004) [hep-ph/0402011].
- [30] F.-K. Guo, C. Hanhart and U.-G. Meißner, Phys. Rev. Lett. **102**, 242004 (2009) [arXiv:0904.3338 [hep-ph]].
- [31] S. Fleming and T. Mehen, Phys. Rev. D **78**, 094019 (2008) [arXiv:0807.2674 [hep-ph]].
- [32] F.-K. Guo, C. Hanhart and U.-G. Meißner, Phys. Lett. B **665**, 26 (2008) [arXiv:0803.1392 [hep-ph]].
- [33] F.-K. Guo, C. Hanhart, G. Li, U.-G. Meißner and Q. Zhao, Phys. Rev. D **83**, 034013 (2011) [arXiv:1008.3632 [hep-ph]].
- [34] S. Dubynskiy, A. Gorsky and M. B. Voloshin, Phys. Lett. B **671**, 82 (2009) [arXiv:0804.2244 [hep-th]].

- [35] F.-K. Guo, P.-N. Shen and H.-C. Chiang, Phys. Rev. D **74**, 014011 (2006) [hep-ph/0604252].
- [36] L. Maiani, F. Piccinini, A. D. Polosa and V. Riquer, Phys. Rev. D **89**, 114010 (2014) [arXiv:1405.1551 [hep-ph]].
- [37] A. Esposito, A. L. Guerrieri, F. Piccinini, A. Pilloni and A. D. Polosa, Int. J. Mod. Phys. A **30**, 1530002 (2015) [arXiv:1411.5997 [hep-ph]].
- [38] L. Maiani, F. Piccinini, A. D. Polosa and V. Riquer, Phys. Rev. D **71**, 014028 (2005) [hep-ph/0412098].
- [39] L. Zhao, W. Z. Deng and S. L. Zhu, Phys. Rev. D **90**, 094031 (2014) [arXiv:1408.3924 [hep-ph]].
- [40] G. Cotugno, R. Faccini, A. D. Polosa and C. Sabelli, Phys. Rev. Lett. **104** (2010) 132005 [arXiv:0911.2178 [hep-ph]].
- [41] F.-K. Guo, J. Haidenbauer, C. Hanhart and U.-G. Meißner, Phys. Rev. D **82**, 094008 (2010) [arXiv:1005.2055 [hep-ph]].
- [42] M. Cleven, Q. Wang, F.-K. Guo, C. Hanhart, U.-G. Meißner and Q. Zhao, Phys. Rev. D **90** (2014) 7, 074039 [arXiv:1310.2190 [hep-ph]].
- [43] C. Z. Yuan, Chin. Phys. C **38**, 043001 (2014) [arXiv:1312.6399 [hep-ex]].
- [44] H. Høgaasen, J. M. Richard and P. Sorba, Phys. Rev. D **73**, 054013 (2006) [hep-ph/0511039].
- [45] F. Buccella, H. Høgaasen, J. M. Richard and P. Sorba, Eur. Phys. J. C **49**, 743 (2007) [hep-ph/0608001].
- [46] T. Guo, L. Cao, M. Z. Zhou and H. Chen, arXiv:1106.2284 [hep-ph].
- [47] H. Høgaasen, E. Kou, J. M. Richard and P. Sorba, Phys. Lett. B **732**, 97 (2014) [arXiv:1309.2049 [hep-ph]].
- [48] F. Stancu, J. Phys. G **37**, 075017 (2010) [arXiv:0906.2485 [hep-ph]].
- [49] S. Weinberg, Phys. Rev. **130**, 776 (1963).
- [50] N. A. Törnqvist, Phys. Lett. B **590**, 209 (2004) [hep-ph/0402237].
- [51] M. B. Voloshin, Phys. Lett. B **579**, 316 (2004) [hep-ph/0309307].
- [52] G. J. Ding, Phys. Rev. D **79**, 014001 (2009) [arXiv:0809.4818 [hep-ph]].
- [53] Q. Wang, C. Hanhart and Q. Zhao, Phys. Rev. Lett. **111**, 132003 (2013) [arXiv:1303.6355 [hep-ph]].
- [54] L. Ma, X. H. Liu, X. Liu and S. L. Zhu, Phys. Rev. D **91**, 034032 (2015) [arXiv:1406.6879 [hep-ph]].
- [55] Q. Wang, M. Cleven, F.-K. Guo, C. Hanhart, U.-G. Meißner, X. G. Wu and Q. Zhao, Phys. Rev. D **89**, 034001 (2014) [arXiv:1309.4303 [hep-ph]].
- [56] M. B. Voloshin, Phys. Rev. D **84**, 031502 (2011) [arXiv:1105.5829 [hep-ph]].
- [57] M. Cleven, F.-K. Guo, C. Hanhart and U.-G. Meißner, Eur. Phys. J. A **47**, 120 (2011) [arXiv:1107.0254 [hep-ph]].
- [58] C. Hanhart, Y. S. Kalashnikova and A. V. Nefediev, Phys. Rev. D **81**, 094028 (2010) [arXiv:1002.4097 [hep-ph]].
- [59] A. A. Filin, A. Romanov, V. Baru, C. Hanhart, Y. S. Kalashnikova, A. E. Kudryavtsev, U.-G. Meißner and A. V. Nefediev, Phys. Rev. Lett. **105**, 019101 (2010) [arXiv:1004.4789 [hep-ph]].
- [60] F.-K. Guo and U.-G. Meißner, Phys. Rev. D **84**, 014013 (2011) [arXiv:1102.3536 [hep-ph]].
- [61] J. Nieves and M. P. Valderrama, Phys. Rev. D **86**, 056004 (2012) [arXiv:1204.2790 [hep-ph]].

- [62] C. Hidalgo-Duque, J. Nieves and M. P. Valderrama, Phys. Rev. D **87**, 076006 (2013) [arXiv:1210.5431 [hep-ph]].
- [63] F.-K. Guo, C. Hidalgo-Duque, J. Nieves and M. P. Valderrama, Phys. Rev. D **88**, 054007 (2013) [arXiv:1303.6608 [hep-ph]].
- [64] S. Fleming, M. Kusunoki, T. Mehen and U. van Kolck, Phys. Rev. D **76**, 034006 (2007) [hep-ph/0703168].
- [65] J. Nieves and M. P. Valderrama, Phys. Rev. D **84**, 056015 (2011) [arXiv:1106.0600 [hep-ph]].
- [66] M. P. Valderrama, Phys. Rev. D **85**, 114037 (2012) [arXiv:1204.2400 [hep-ph]].
- [67] V. Baru, E. Epelbaum, A. A. Filin, F.-K. Guo, H.-W. Hammer, C. Hanhart, U.-G. Meißner and A. V. Nefediev, Phys. Rev. D **91**, 034002 (2015) [arXiv:1501.02924 [hep-ph]].
- [68] M. H. Alhakami and M. C. Birse, Phys. Rev. D **91**, 054019 (2015) [arXiv:1501.06750 [hep-ph]].
- [69] E. Braaten, arXiv:1503.04791 [hep-ph].
- [70] N. A. Törnqvist, Z. Phys. C **61**, 525 (1994) [hep-ph/9310247].
- [71] Y. J. Zhang, H. C. Chiang, P. N. Shen and B. S. Zou, Phys. Rev. D **74**, 014013 (2006) [hep-ph/0604271].
- [72] C. E. Thomas and F. E. Close, Phys. Rev. D **78**, 034007 (2008) [arXiv:0805.3653 [hep-ph]].
- [73] X. Liu, Z. G. Luo, Y. R. Liu and S. L. Zhu, Eur. Phys. J. C **61**, 411 (2009) [arXiv:0808.0073 [hep-ph]].
- [74] Y. R. Liu and Z. Y. Zhang, Phys. Rev. C **80**, 015208 (2009) [arXiv:0810.1598 [hep-ph]].
- [75] R. Molina and E. Oset, Phys. Rev. D **80**, 114013 (2009) [arXiv:0907.3043 [hep-ph]].
- [76] I. W. Lee, A. Faessler, T. Gutsche and V. E. Lyubovitskij, Phys. Rev. D **80**, 094005 (2009) [arXiv:0910.1009 [hep-ph]].
- [77] Z. F. Sun, J. He, X. Liu, Z. G. Luo and S. L. Zhu, Phys. Rev. D **84**, 054002 (2011) [arXiv:1106.2968 [hep-ph]].
- [78] S. Ohkoda, Y. Yamaguchi, S. Yasui, K. Sudoh and A. Hosaka, Phys. Rev. D **86**, 014004 (2012) [arXiv:1111.2921 [hep-ph]].
- [79] A. Ozpineci, C. W. Xiao and E. Oset, Phys. Rev. D **88**, 034018 (2013) [arXiv:1306.3154 [hep-ph]].
- [80] J. He, X. Liu, Z. F. Sun and S. L. Zhu, Eur. Phys. J. C **73**, 2635 (2013) [arXiv:1308.2999 [hep-ph]].
- [81] F. Aceti, M. Bayar, J. M. Dias and E. Oset, Eur. Phys. J. A **50**, 103 (2014) [arXiv:1401.2076 [hep-ph]].
- [82] F. Aceti, M. Bayar, E. Oset, A. Martinez Torres, K. P. Khemchandani, J. M. Dias, F. S. Navarra and M. Nielsen, Phys. Rev. D **90**, 016003 (2014) [arXiv:1401.8216 [hep-ph]].
- [83] L. Zhao, L. Ma and S. L. Zhu, Phys. Rev. D **89**, 094026 (2014) [arXiv:1403.4043 [hep-ph]].
- [84] X. Liu, Y. R. Liu, W. Z. Deng and S. L. Zhu, Phys. Rev. D **77**, 094015 (2008) [arXiv:0803.1295 [hep-ph]].
- [85] G. J. Ding, W. Huang, J. F. Liu and M. L. Yan, Phys. Rev. D **79**, 034026 (2009) [arXiv:0805.3822 [hep-ph]].
- [86] M. T. Li, W. L. Wang, Y. B. Dong and Z. Y. Zhang, Commun. Theor. Phys. **63**, 63 (2015).
- [87] F. Gross, *Relativistic Quantum Mechanics and Field Theory*, John Wiley & Sons, New York, 1993.
- [88] T. E. O. Ericson and G. Karl, Phys. Lett. B **309**, 426 (1993).

- [89] N. A. Törnqvist, Phys. Rev. Lett. **67**, 556 (1991).
- [90] M. Albaladejo, F.-K. Guo, C. Hidalgo-Duque, J. Nieves and M. P. Valderrama, arXiv:1504.00861 [hep-ph].
- [91] C. Hanhart, J. R. Peláez and G. Ríos, Phys. Lett. B **739**, 375 (2014) [arXiv:1407.7452 [hep-ph]].
- [92] Q. Wang, Phys. Rev. D **89**, 114013 (2014) [arXiv:1403.2243 [hep-ph]].